

# BB MODE SPECTRUM OF CMB AND INFLATION

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## Abstract

Quantum effect on the BB-mode correlation spectrum of Cosmic Microwave Background for several inflation models is studied with the BICEP2/Keck Array and Planck joint data. The results do not rule out either single or multi field models of slow-roll inflation. The quantum effect is found more prominent for the inflation models with larger values of tensor-to-scalar ratio and smaller values of tensor spectral index.

## 1 Introduction

Inflation, a sudden exponential expansion which occurred for a brief period in the very early stage of the universe, is the highly considered scenario to overcome some of the shortcomings of the standard model of cosmology. There exist a large number of inflation models by now which include single as well as multi field inflation models [1, 2, 3, 4, 5]. All the models may agree that quantum fluctuations during inflation led to the scalar and tensor perturbations. The scalar perturbations represent the density fluctuations which seeded the galaxy formation in the universe that we observe today, while the tensor perturbations represent primordial gravitational waves (GWs). The primordial GWs were generated during the inflation by strong and variable gravitational field of early universe via a mechanism known as parametric amplification of the zero-point quantum fluctuations which transforms the initial vacuum state with no particle into a multi-particle quantum state [6, 7], called squeezed vacuum state [8]. Therefore the primordial GWs are supposed to be found in the squeezed vacuum state [9]. Further, it is believed that the tensor fluctuations have left their own signatures on the cosmic microwave background (CMB) [10, 11, 12]. Hence the squeezing effect may also be expected to reflect on the BB-mode correlation angular power spectrum of CMB [13].

Single field slow-roll inflation predicts almost Gaussian distribution of adiabatic density perturbations with an almost scale invariant spectrum, whereas multi field inflation generate non-adiabatic perturbations giving rise to non-Gaussianity. According to the recent results from Planck mission the "primordial non-Gaussianity is small" [14] which tempts to conclude that single field inflation models are favorable over multi field. However, one of the attempts [15, 16, 17] to explain the hemispherical asymmetry in the CMB sky, hinted by WMAP [18, 19, 20] and confirmed by Planck 2013 and 2015 data [21, 22], indicates that a single field slow roll inflation cannot produce such an asymmetry without violating the homogeneity of the universe. At same

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time multi scalar field (most probably inflaton and curvaton) during inflation can produce such an asymmetry without violating the homogeneity [17]. The goal of the paper is not study the Gaussianity issue, however the aforementioned studies show that it is necessary to address and resolve the issue of single field or multi field of inflation with alternative approaches than the Gaussianity or non-Gaussianity test. Therefore, the present work is aimed to explore whether the single field and multi field models of inflation issue can be resolved by considering the GWs in the squeezed vacuum state and then its effect on the  $BB$  mode power spectrum of CMB with the BICEP2/Keck Array and Planck collaboration data [23].

## 2 GW in squeezed vacuum state

The perturbed form of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric for flat universe is given by

$$ds^2 = S^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \quad (1)$$

where  $h_{ij}$  is a transverse-traceless perturbation with

$$\partial_i h^{ij} = 0, \quad \delta^{ij} h_{ij} = 0,$$

and  $|h_{ij}| \ll \delta_{ij}$ , where  $\delta_{ij}$  is the space metric,  $S$  is the scale factor and  $\tau$  is the conformal time.

The GW  $h_{ij}(\mathbf{x}, \tau)$  can be decomposed into a collection of Fourier modes as

$$h_{ij}(\mathbf{x}, \tau) = \frac{C}{(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{+\infty} \frac{d^3 \mathbf{k}}{\sqrt{2k}} \sum_{p=1}^2 [h_k^{(p)}(\tau) c_k^{(p)} \varepsilon_{ij}^{(p)}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} + h_k^{(p)*}(\tau) c_k^{(p)\dagger} \varepsilon_{ij}^{(p)*}(\mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{x}}],$$

where  $C = \sqrt{16\pi} l_{pl}$  and  $l_{pl} = \sqrt{G}$  is the Planck length.

The polarization states  $\varepsilon_{ij}^{(p)}$ ,  $p = 1, 2$  are transverse-traceless and symmetric which obey the conditions  $\varepsilon_{ij}^{(p)} \delta^{ij} = 0$ ,  $\varepsilon_{ij}^{(p)} k^i = 0$ ,  $\varepsilon_{ij}^{(p)} \varepsilon^{(p')ij} = 2\delta_{pp'}$ ,  $\varepsilon_{ij}^{(p)}(-\mathbf{k}) = \varepsilon_{ij}^{(p)}(\mathbf{k})$ . These are linear polarizations, called the plus (+) polarization and cross (×) polarization. The operators  $c_k^\dagger$  and  $c_k$  follow the conditions,  $[c_k^{(p)}, c_{k'}^{(p')\dagger}] = \delta_{pp'} \delta^3(k - k')$ ,  $[c_k^{(p)}, c_{k'}^{(p')}] = [c_k^{(p)\dagger}, c_{k'}^{(p')\dagger}] = 0$  and are governed by the Heisenberg equations given by

$$\frac{d}{d\tau} c_{\mathbf{k}}^\dagger(\tau) = -i[c_{\mathbf{k}}^\dagger(\tau), H], \quad (2)$$

$$\frac{d}{d\tau} c_{\mathbf{k}}(\tau) = -i[c_{\mathbf{k}}(\tau), H]. \quad (3)$$

The initial vacuum state  $|0\rangle$  is defined as

$$c_{\mathbf{k}}^p |0\rangle = 0.$$

The Bogoliubov transformations for the operators  $c_{\mathbf{k}}^\dagger$  and  $c_{\mathbf{k}}$  are

$$c_{\mathbf{k}}^\dagger(\tau) = u_{\mathbf{k}}^*(\tau) c_{\mathbf{k}}^\dagger(0) + v_{\mathbf{k}}^*(\tau) c_{\mathbf{k}}(0), \quad (4)$$

$$c_{\mathbf{k}}(\tau) = u_{\mathbf{k}}(\tau) c_{\mathbf{k}}(0) + v_{\mathbf{k}}(\tau) c_{\mathbf{k}}^\dagger(0), \quad (5)$$

where  $c_{\mathbf{k}}^\dagger(0)$  is the initial value of the creation operator and  $c_{\mathbf{k}}(0)$  is that for the annihilation operator. The complex functions  $u_k(\tau)$  and  $v_k(\tau)$  satisfy the following condition

$$|u_k|^2 - |v_k|^2 = 1.$$

The mode functions  $h_k(\tau)$  with scale factor  $S(\tau)$  can be taken as

$$h_k^{(p)} = \frac{\psi_k^{(p)}}{S}. \quad (6)$$

The mode functions can have the following form <sup>1</sup>.

$$\psi_k(\tau) = u_k(\tau) + v_k^*(\tau), \quad (7)$$

which satisfies the equation of motion

$$\psi_k'' + \left(k^2 - \frac{S''}{S}\right) \psi_k = 0. \quad (8)$$

Two-point correlation function of the Fourier modes gives the power spectrum of tensor perturbations and can be written as

$$\langle h_k h_{k'}^* \rangle = \frac{2\pi^2}{k^3} P_T(k) \delta^3(\mathbf{k} - \mathbf{k}'), \quad (9)$$

where  $P_T$  is the gravitational wave power spectrum and  $\langle \rangle$  denotes ensemble average.

The gravitational wave can be written in terms of the mode function and the annihilation and creation operators, taking the contribution from each polarization to be the same,

$$h(\mathbf{x}, \tau) = \frac{C}{S(\tau)(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{+\infty} d^3\mathbf{k} [\psi_k(\tau) c_k + \psi_k^*(\tau) c_k^\dagger] e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (10)$$

The squeezed vacuum state can be defined as [24]

$$|\xi\rangle = Z(\xi)|0\rangle, \quad (11)$$

where the squeezing operator  $Z(\xi)$  can be written as

$$Z(\xi) = \exp \left[ \frac{1}{2} \xi^* b^2 - \frac{1}{2} \xi b^{\dagger 2} \right], \quad (12)$$

where  $\xi = r_s e^{i\gamma}$ ,  $r_s$  and  $\gamma$  are respectively the squeezing parameter and squeezing angle. The action of the squeezing operator  $Z$  on the annihilation and creation operators gives

$$\begin{aligned} Z^\dagger(\xi) b Z(\xi) &= b \cosh r_s - b^\dagger e^{i\gamma} \sinh r_s, \\ Z^\dagger(\xi) b^\dagger Z(\xi) &= b^\dagger \cosh r_s - b e^{-i\gamma} \sinh r_s. \end{aligned} \quad (13)$$

The functions  $u_k(\tau)$  and  $v_k(\tau)$  in Eq.(7) can be represented in terms of three real functions: the rotation angle  $\theta_s$ , the squeezing parameter  $r_s$  and squeezing angle  $\gamma$  as

$$\begin{aligned} u_k &= e^{i\theta_s} \cosh r_s, \\ v_k &= e^{-i(\theta_s - 2\gamma)} \sinh r_s. \end{aligned} \quad (14)$$

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<sup>1</sup>Since the contribution from each polarization is same, here onward, we drop the superscript  $(p)$

Using Eqns.(2) and (13), the tensor power spectrum for GWs in the squeezed vacuum state is obtained as,

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'}^* \rangle = \frac{C^2}{S^2} \left[ (1 + 2 \sinh^2 r_s) |\psi_k|^2 + \frac{1}{2} \sinh 2r_s (\psi_k^2 e^{i\gamma} + \psi_k^{*2} e^{-i\gamma}) \right] \delta^3(\mathbf{k} - \mathbf{k}'). \quad (15)$$

From Eq.(9) and Eq.(15), we get the tensor power spectrum in the squeezed vacuum state as

$$P_T(k) = \frac{k^3}{2\pi^2} \frac{C^2}{S^2} \left[ (1 + 2 \sinh^2 r_s) |\psi_k|^2 + \frac{1}{2} \sinh 2r_s (\psi_k^2 e^{i\gamma} + \psi_k^{*2} e^{-i\gamma}) \right]. \quad (16)$$

For the quasi de Sitter universe, during inflation the conformal time and the scale factor are related as  $S(\tau) = \frac{-1}{H\tau(1-\epsilon)}$ , where  $\epsilon = \frac{m_{pl}^2}{2} \left( \frac{V'}{V} \right)^2$  and  $V$  is the potential of the scalar field. For small value of the slow-roll parameter  $\epsilon$ ,  $\vartheta = \frac{3}{2} + \epsilon$ , and  $n_T = -2\epsilon = 3 - 2\vartheta$ .

For constant  $\epsilon$ , the equation of motion can be written as [25]

$$\psi_k'' + \left[ k^2 - \frac{1}{\tau^2} \left( \vartheta^2 - \frac{1}{4} \right) \right] \psi_k = 0. \quad (17)$$

The general solution for the above equation (Eq.(17)) is

$$\psi_k(\tau) = \sqrt{-\tau} [C_1(k) \mathbb{H}_\vartheta^{(1)}(-k\tau) + C_2(k) \mathbb{H}_\vartheta^{(2)}(-k\tau)], \quad (18)$$

where  $\mathbb{H}_\vartheta^{(1)}$  and  $\mathbb{H}_\vartheta^{(2)}$  are the Hankel functions of the first and second kind, and  $C_1$  and  $C_2$  are the integration constants.

Within the horizon ( $k \gg SH$ ), the modes can be approximated using the flat spacetime solutions as

$$\psi_k^0(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}.$$

Using the above solution, the constants of integration become

$$\begin{aligned} C_1(k) &= \frac{\sqrt{\pi}}{2} \exp \left[ i \left( \vartheta + \frac{1}{2} \right) \left( \frac{\pi}{2} \right) \right], \\ C_2(k) &= 0. \end{aligned} \quad (19)$$

Hence for long wavelength limiting case ( $k \ll SH$ ), Eq.(18) gives

$$\psi_k(\tau) = e^{i(\vartheta - \frac{1}{2})(\frac{\pi}{2})} 2^{\vartheta - \frac{3}{2}} \frac{\Gamma(\vartheta)}{\Gamma(\frac{3}{2})} \frac{1}{\sqrt{2k}} (-k\tau)^{\frac{1}{2} - \vartheta}. \quad (20)$$

Using Eq.(20) in Eq.(16), the tensor power spectrum for GWs in the superhorizon limit ( $k \ll SH$ ) is,

$$P_T(k) = C^2 \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{SH} \right)^{3-2\vartheta} \left[ 1 + 2 \sinh^2 r_s + \sinh 2r_s \cos \left( \gamma + \left( \vartheta - \frac{1}{2} \right) \pi \right) \right].$$

By taking  $A_T(k_0) = C^2 \left( \frac{H_{k_0}}{2\pi} \right)^2$  with  $H_{k_0}$ , the Hubble parameter, at  $SH = k_0$  during the inflation,  $k_0$  is the pivot wavenumber, the tensor power spectrum in terms of the tensor spectral index  $n_T$  is obtained as

$$P_T(k) = A_T(k_0) \left( \frac{k}{k_0} \right)^{n_T} \left[ 1 + 2 \sinh^2 r_s + \sinh 2r_s \cos \left( \gamma + (2 - n_T) \frac{\pi}{2} \right) \right],$$

which is the tensor power spectrum of GWs in the squeezed vacuum state.

All GW modes start in the same vacuum state with  $r_s = 0$  initially. However, as mentioned earlier, quantum mechanical evolution under parametric influence transforms the initial vacuum state into strongly squeezed vacuum state [26]. The variance of the mode's phase is strongly squeezed whereas the variance of its amplitude is being enhanced so that the uncertainty product remains intact. The parameter of squeezing grows all the way up in the amplifying regime and can vary from zero in the vacuum state up to a very large value by the end of the amplifying regime. Its value for the present epoch is  $1.2 \times 10^{-2}$  [27]. For the inflationary period, it is up to 1 ( $0 \leq r_s \leq 1$ ) which, we use for the present study.

### 3 Slow-roll inflationary scenario

In the simplest inflationary scenario, a homogeneous scalar field known as inflaton  $\phi$ , drives the accelerated expansion of early universe.

The equation of motion for the inflaton can be written as

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (21)$$

where the Hubble parameter is determined by the energy density of the field,

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V,$$

so that the Friedmann equation becomes

$$H^2 = \frac{1}{3m_{pl}^2} \left( \frac{1}{2}\dot{\phi}^2 + V \right). \quad (22)$$

Under the slow-roll limiting condition the Friedmann equation takes the following form

$$H^2 \simeq \frac{V}{3m_{pl}^2}. \quad (23)$$

This can be characterized with the slow-roll parameters which are defined as

$$\begin{aligned} \epsilon &\equiv \frac{m_{pl}^2}{2} \left( \frac{V'}{V} \right)^2, \\ \eta &\equiv m_{pl}^2 \left( \frac{V''}{V} \right), \end{aligned} \quad (24)$$

and so on. Inflation lasts as long as  $\epsilon \ll 1$  and  $|\eta| \ll 1$  and it keeps the Hubble rate nearly constant. The slow-roll parameters can be used to study the fluctuations generated during inflation.

The parameter that measures the relative strength of the tensor spectrum to the scalar spectrum is given by

$$r \equiv \frac{P_T(k)}{P_S(k)} \simeq 16\epsilon, \quad (25)$$

which is known as the tensor-to-scalar ratio and is very useful to distinguish different models of inflation. For the present study the scalar power spectrum is taken to be  $P_S = 2.43 \times 10^{-9}$ .

## 4 Slow-roll inflation models

In this section, we discuss several slow-roll inflation models briefly for which the upper bound for the tensor-to-scalar ratio is  $r < 0.07$  and lower limit is  $r \simeq \mathcal{O}(10^{-3})$  and are constrained with various CMB observations [28]. We compute the slow roll parameters, tensor spectral index and tensor power spectrum for the following slow-roll inflation models.

### 4.1 Natural inflation model

In this model the inflaton is considered to be the pseudo-Nambu Goldstone boson which arises whenever global symmetry is broken [29, 30].

The corresponding potential of the inflaton is given by

$$V(\phi) = M^4 \left[ 1 + \cos \left( \frac{\phi}{f} \right) \right], \quad (26)$$

where  $f/m_{pl} = 10^2$  is the energy scale at which symmetry is broken,  $M/m_{pl} = 10^{-2}$ .

The obtained parameters for the Natural inflation model are

$$\begin{aligned} r &= 2.06 \times 10^{-2}, \\ \epsilon &= 1.29 \times 10^{-3}, \\ \eta &= 1.24 \times 10^{-3}, \\ n_T &= -2.58 \times 10^{-3}. \end{aligned}$$

The initial tensor power spectrum for the model is obtained as  $P_T = 5.027 \times 10^{-11}$ .

### 4.2 Arctan inflation model

This model is often studied as a toy model [31, 32]. In the present study, we consider a large field model which starts at a large value and then evolves to a minimum at the origin.

The potential for the model is

$$V(\phi) = M^4 \left[ 1 - \arctan \left( \frac{\phi}{\mu} \right) \right], \quad (27)$$

where  $\mu/m_{pl} = 10^{-2}$  is a free parameter which characterizes the typical vacuum expectation value at which inflation takes place,  $M/m_{pl} = 10^{-3}$ .

Thus the parameters for the Arctan inflation model are obtained as

$$\begin{aligned} r &= 1.38 \times 10^{-2}, \\ \epsilon &= 8.62 \times 10^{-4}, \\ \eta &= 3.0 \times 10^{-2}, \\ n_T &= -1.72 \times 10^{-3}. \end{aligned}$$

The initial tensor power spectrum for the model is obtained as,  $P_T = 3.35 \times 10^{-11}$ .

### 4.3 Inverse monomial inflation model

This model is studied in the context of quintessential inflation [33, 34, 35]; the inflaton need not necessarily decay and reheating arises naturally even when the potential does not have a global minimum, radiation is created via gravitational particle production.

The inflaton potential for the model is

$$V(\phi) = M^4 \left( \frac{\phi}{m_{pl}} \right)^{-p}, \quad (28)$$

where  $p$  is a positive parameter,  $M/m_{pl} = 10^{-1}$ .

The corresponding calculated parameters are:

$$\begin{aligned} r &= 2.0 \times 10^{-3}, \\ \epsilon &= 1.25 \times 10^{-4}, \\ \eta &= 3.33 \times 10^{-4}, \\ n_T &= -2.50 \times 10^{-4}. \end{aligned}$$

The corresponding initial tensor power spectrum for this model is,  $P_T = 4.86 \times 10^{-12}$ .

### 4.4 Loop inflation model

In this scenario, the flatness of the inflaton potential is altered by symmetry breaking which produces quantum radiative corrections in which one loop order correction takes the form of a logarithmic function [36, 37, 38].

The potential for the loop inflation model can be written as

$$V(\phi) = M^4 \left[ 1 + \alpha \ln \left( \frac{\phi}{m_{pl}} \right) \right], \quad (29)$$

where  $\alpha = g^2/16\pi^2$  tunes the strength of radiative effects,  $M = 10^{16}$  GeV.

The corresponding calculated parameters are

$$\begin{aligned} r &= 4.34 \times 10^{-2}, \\ \epsilon &= 3.09 \times 10^{-3}, \\ \eta &= -2.06 \times 10^{-2}, \\ n_T &= -6.18 \times 10^{-3}. \end{aligned}$$

The initial tensor power spectrum is obtained as,  $P_T = 1.2 \times 10^{-10}$ .

### 4.5 Coleman-Weinberg inflation model

The potential in this scenario is introduced in the context of spontaneous symmetry breaking generated by radiative corrections [39, 40, 41].

The potential for the model is

$$V(\phi) = M^4 \left[ 1 + \alpha \left( \frac{\phi}{\sigma} \right)^4 \ln \left( \frac{\phi}{\sigma} \right) \right], \quad (30)$$

where  $\alpha = 4e$ ,  $M = 10^{16}$  GeV,  $\sigma = 10m_{pl}$  sets the typical vacuum expectation value at which inflation takes place.

The parameters are obtained as

$$\begin{aligned} r &= 7.77 \times 10^{-3}, \\ \epsilon &= 4.86 \times 10^{-4}, \\ \eta &= -4.42 \times 10^{-2}, \\ n_T &= -9.72 \times 10^{-4}. \end{aligned}$$

The corresponding initial tensor power spectrum is,  $P_T = 1.89 \times 10^{-11}$ .

#### 4.6 Quadratic chaotic inflation model with radiative corrections

This model is a simple quadratic chaotic inflation model [42] studied under the assumption that the scalar field interacts with the fermion field thus leading to the quantum radiative correction which takes the form of a logarithmic function [43].

The potential with radiative correction is

$$V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{g^4}{16\pi^2}\phi^4 \ln\left(\frac{\phi}{m_{pl}}\right), \quad (31)$$

where  $g$  is the Yukawa coupling and  $m = 3.44 \times 10^{12}$  GeV.

The associated parameters are obtained as

$$\begin{aligned} r &= 2.98 \times 10^{-2}, \\ \epsilon &= 1.86 \times 10^{-3}, \\ \eta &= 1.86 \times 10^{-3}, \\ n_T &= -3.72 \times 10^{-3}. \end{aligned}$$

The initial tensor power spectrum for this model is obtained as,  $P_T = 7.25 \times 10^{-11}$ .

#### 4.7 Hybrid inflation model

This is a multi-scalar field [44, 45] model where  $\phi$  field drives the inflation and the symmetry breaking of  $\sigma$  field triggers the end of inflation.

The potential of hybrid inflation model is

$$V = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\sigma^2, \quad (32)$$

where  $M = 1.21 \times 10^{16}$  GeV,  $m = 3.65 \times 10^{11}$  GeV,  $\lambda = 1$ ,  $g = 8 \times 10^{-4}$ .

The various obtained parameters of the model are

$$\begin{aligned} r &= 4.24 \times 10^{-3}, \\ \epsilon &= 2.65 \times 10^{-4}, \\ \eta &= 1.47 \times 10^{-4}, \\ n_T &= -5.3 \times 10^{-4}. \end{aligned}$$

The initial tensor power spectrum for the model is obtained as,  $P_T = 1.03 \times 10^{-11}$ .



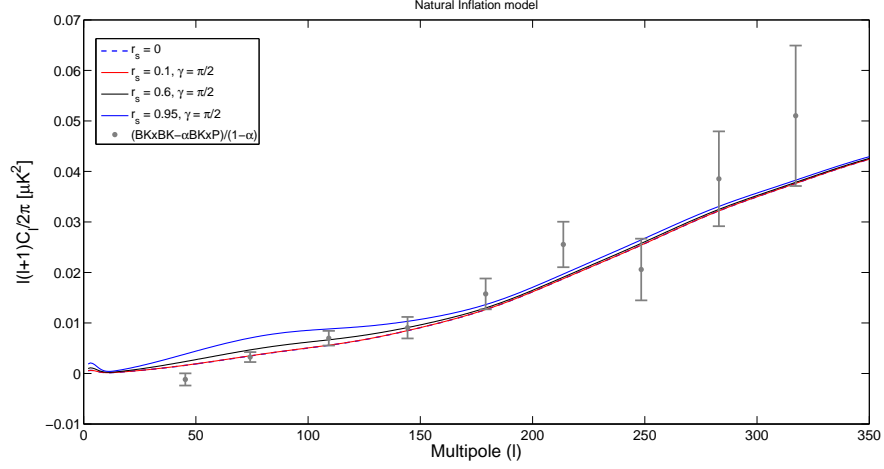


Figure 1:  $BB$ -mode correlation angular power spectrum of CMB for the Natural inflation model for various values of squeezing parameter with joint data of BICEP2/ Keck Array and Planck.

## 5 $BB$ -mode correlation angular power spectrum

The  $BB$ -mode correlation angular power spectrum of CMB in terms of the multipole moments  $l$  is given by [46, 47]

$$C_l^{BB} = (4\pi)^2 \int dk k^2 P_T(k) \left| \int_0^{\tau_0} d\tau g(\tau) h_k(\tau) \left\{ (8x + 2x^2 \partial_x) \frac{j_l(x)}{x^2} \right\}_{x=k(\tau_0-\tau)} \right|^2$$

where  $g(\tau) = \dot{\kappa} e^{-\kappa}$  is the probability distribution of the last scattering with the optical depth  $\kappa$  and  $j_l(x)$  is the spherical Bessel function.

The  $BB$ -mode correlation angular power spectrum of CMB for the aforementioned slow-roll inflationary models is obtained. The angular power spectrum of CMB for the inflation models is generated using the CAMB code with  $n_T$  corresponding to each model. For all inflation models, the optical depth is taken as  $\kappa = 0.08$ , the pivot wave number for tensor mode and scalar mode respectively taken as  $k_0 = 0.002 \text{ Mpc}^{-1}$  and  $k_0 = 0.05 \text{ Mpc}^{-1}$ . The obtained results are compared with the joint analysis data of BICEP2/Keck Array and Planck. The limit  $(BK \times BK - \alpha BK \times P)/(1 - \alpha)$  at  $\alpha = \alpha_{fid} = 0.04$  is taken after the subtraction of the dust contribution (which is 0.04 times as much in the BICEP2 band as it is in the Planck 353 GHz band).

The obtained  $BB$ -mode correlation angular power spectrum of CMB for the different inflation models with various values of squeezing parameter are studied with the BICEP2/Keck Array and Planck joint data and results are given in Figs.1, 2, 3, 4, 5, 6 and 7.

## 6 Discussion and conclusion

The  $BB$  mode correlation angular power spectrum of CMB for several slow-roll inflation models is studied by considering the primordial gravitational in the squeezed vacuum state. The obtained  $BB$  mode correlation angular power spectra for different inflation models are found

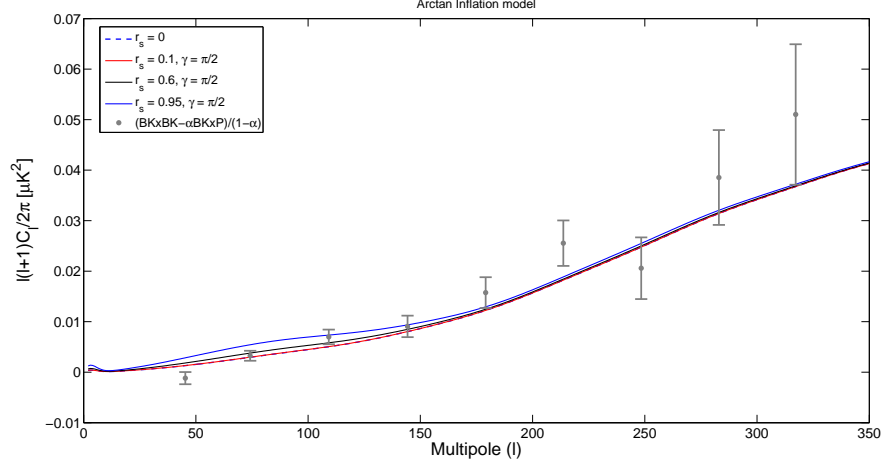


Figure 2:  $BB$ -mode correlation angular power spectrum of CMB for the Arctan inflation model for various values of squeezing parameter with joint data of BICEP2/ Keck Array and Planck.

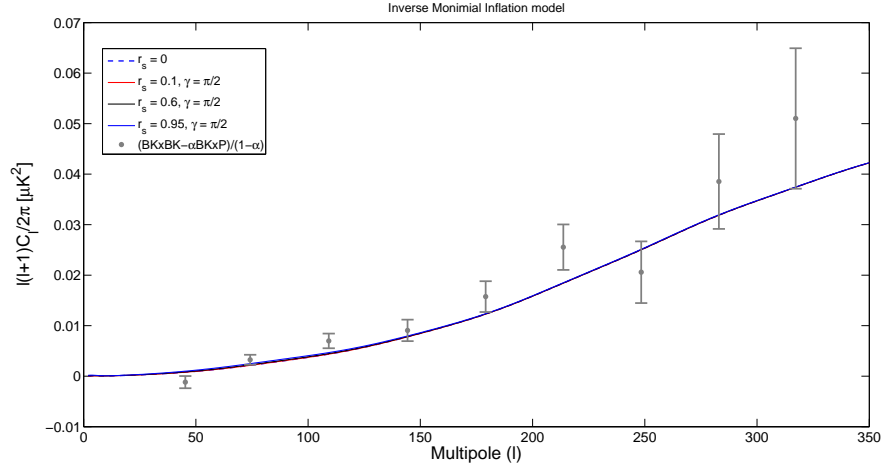


Figure 3:  $BB$ -mode correlation angular power spectrum of CMB for the Inverse monomial inflation model for various values of squeezing parameter with joint data of BICEP2/ Keck Array and Planck.

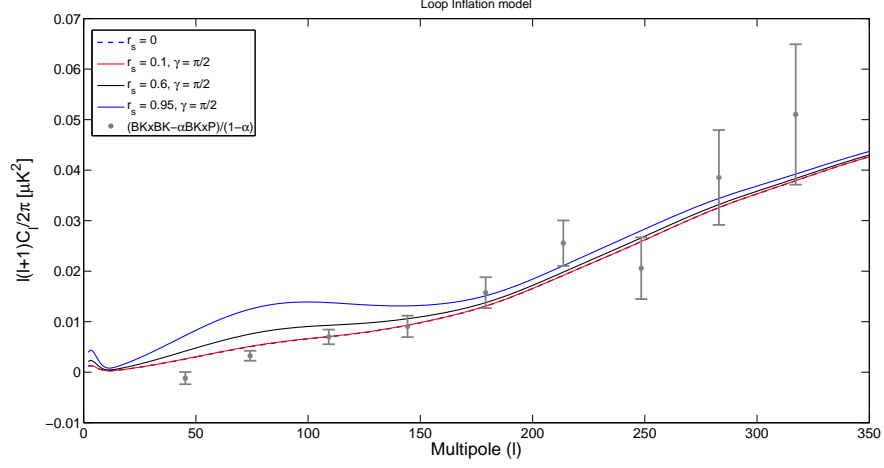


Figure 4:  $BB$ -mode correlation angular power spectrum of CMB for the Loop inflation model for various values of squeezing parameter with joint data of BICEP2/ Keck Array and Planck.

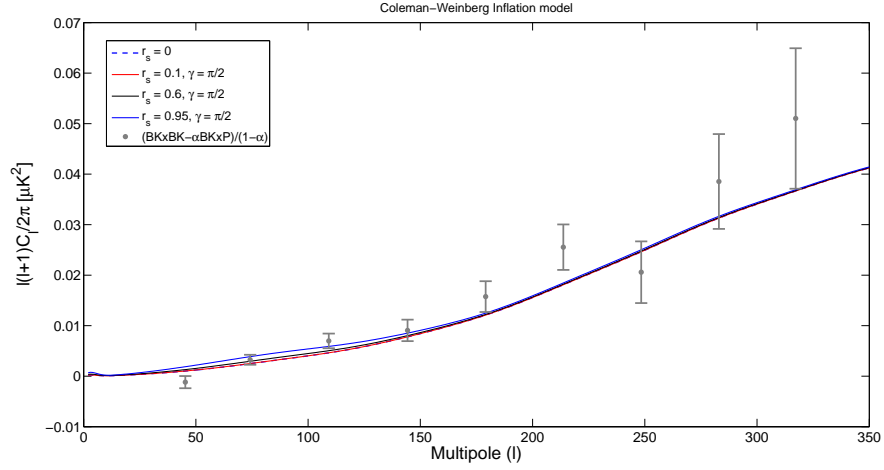


Figure 5:  $BB$ -mode correlation angular power spectrum of CMB for the Coleman-Weinberg inflation model for various values of squeezing parameter with joint data of BICEP2/ Keck Array and Planck.

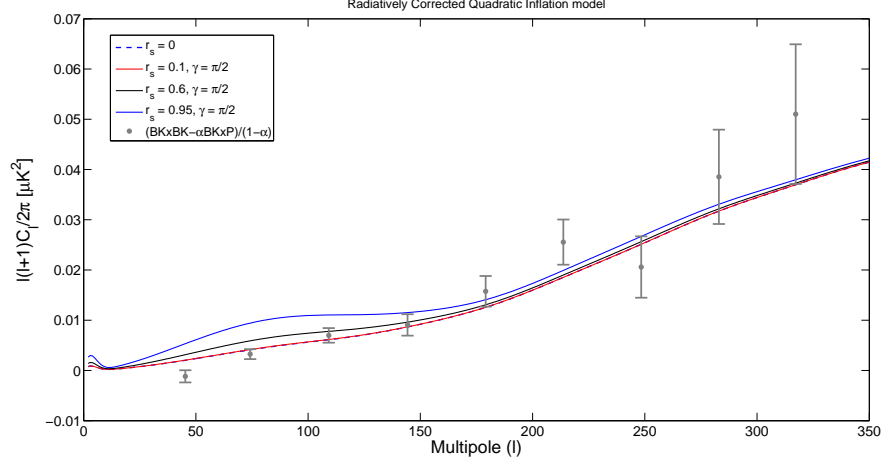


Figure 6:  $BB$ -mode correlation angular power spectrum of CMB for the Quadratic chaotic inflation model with radiative corrections for various values of squeezing parameter with joint data of BICEP2/ Keck Array and Planck.

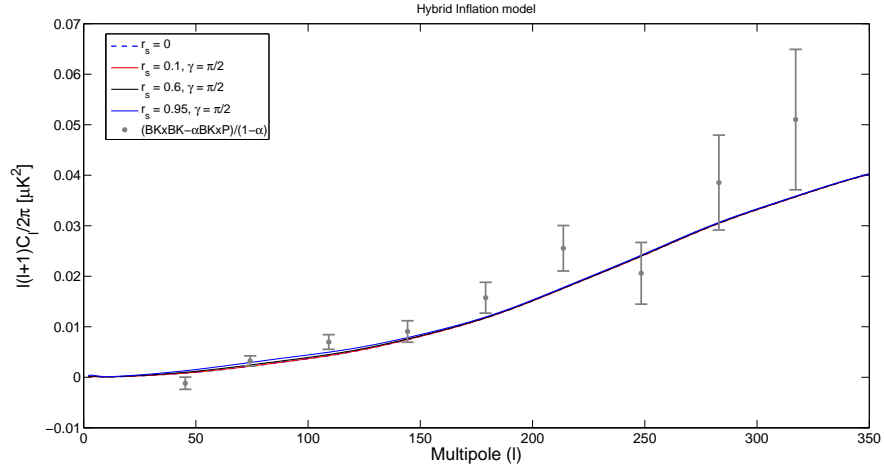


Figure 7:  $BB$ -mode correlation angular power spectrum of CMB for the Hybrid inflation model for various values of squeezing parameter with joint data of BICEP2/ Keck Array and Planck.

within the constraint of recent collaboration data of BICEP2/ Keck Array at 150 GHz and Planck 353 GHz. Note that higher multipoles (smaller angles) are ignored for the analysis due to contamination from lensing effect. It can be observed that the  $BB$  mode angular spectrum gets enhanced with increase in squeezing parameter value for all the inflation models. Further, larger the deviation from scale invariance, stronger is the squeezing effect. That is, for models with larger values of tensor-to-scalar ratio and smaller values of tensor spectral index, the squeezing effect is found more prominent. These show the role of quantum phenomena on the primordial GWs and consequently on the  $BB$  mode power spectrum of CMB.

Note that, all the inflationary models that are considered in the present work are large single field models except the Hybrid inflation model which is a multi-field model. The recent results from Planck mission favour single field inflation models. The attempts that are made to explain the observed hemispherical symmetry in the CMB sky with single field slow roll inflation turns out to be unsuccessful because it cannot produce such an asymmetry without violating the homogeneity of the universe but multi field can generate such asymmetry. The present work made an attempt to resolve the tension between single field and multi field models of inflation based on the  $BB$  mode spectrum of CMB. However, the results of the present study on  $BB$  mode correlation angular power spectrum of CMB for various slow-roll models of inflation with the current joint data of BICEP2/ Keck Array and Planck also do not rule out either single or multi field scalar field models of inflation. From these studies it may be concluded that studies at very fundamental level is required to understand the multi or single field issue of inflation models. Various cosmological observations and further study may resolve this important issues in nearby future.

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